

Comment on “Next-to-leading order forward hadron production in the small- x regime: rapidity factorization” arXiv:1403.5221 by Kang *et al.*

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In a recent paper [1], Kang *et al.* proposed the so-called “rapidity factorization” for the single inclusive forward hadron production in pA collisions. We point out that the leading small- x logarithm was mis-identified in this paper, and hence the newly added next-to-leading order correction term is unjustified and should be absent in view of the small- x factorization.

Single inclusive hadron production in the forward pA collisions is one of the simplest and most interesting processes which can probe the onset of gluon saturation in dense nuclear targets. The leading order formula for this process was derived in Ref. [2] in 2002. The next-to-leading order (NLO) corrections were calculated in the small- x factorization formalism in Refs. [3, 4] a few years ago. In a recent publication [1], the NLO result was re-derived following the same method in Ref. [3], and reproduced what have been computed (including the collinear factorization for the parton distribution and fragmentation functions), except for the small- x factorization part. In lieu of the small- x factorization scheme used in Ref. [3], Ref. [1] proposed the so-called “rapidity factorization”. We disagree with the rapidity factorization argument used in Ref. [1] which leads to a new NLO correction. We believe that this comes from a mis-identification of leading logarithms at small- x in their calculations.

First of all, as a general remark, in the final result of Ref. [1], the hadronic kinematic variable $Y = \ln\left(\frac{s}{m_p^2}\right)$ appears. Here, s and m_p are the center of mass energy squared and the proton mass. This violates the generic factorization principle in parton physics, where the hadronic cross section should be written as a convolution of parton distribution and the partonic cross section. The latter does not depend on the hadrons’ momenta. If this does not hold, it means that there is no factorization at all. As we have already demonstrated the factorization for this process in Ref. [3], the hadronic variable Y never enters in the factorization formula for single inclusive hadron production. It appears in the result of Ref. [1] due to a mis-identification of the leading logarithms of small- x resummation. We will elaborate more on this in the following. In addition, the appearance of the proton mass in the result of [1] cast strong suspicion from the perturbative calculation point of view, since proton mass has normally be regarded as nonperturbative scale. How proton mass enters in a perturbative calculations needs further justification.

The new NLO correction in the so-called “rapidity factorization” arises from the mis-identification of the large logarithms associated with small- x evolution in Ref.[1]. In the following, we show the correct evaluation of the large logarithms associated with the small- x physics, and demonstrate how to obtain the consistent result in the spirit of factorization. These derivations have been clearly shown in Ref. [3, 5]. To emphasize the conceptual difference, we elaborate these arguments step by step as follows.

We take the quark channel contribution as an example. The leading contribution can be formulated as the quark scattering on nucleus target and fragmenting into a final state hadron, and the cross section is written as

$$\frac{d\sigma(pA \rightarrow h + X)}{dyd^2p_\perp} = \int q(x_p) \otimes D_q(z) \otimes \mathcal{F}_{x_g}(r_\perp) , \quad (1)$$

where $x_p = k_\perp e^y/\sqrt{s}$, $k_\perp = p_\perp/z$, and $x_g = k_\perp e^{-y}/\sqrt{s}$ representing the x -value at which the dipole amplitude is evaluated for the quark production.

Although it was not explicitly written, from their calculation, the factorization that Ref. [1] claimed seems take the following form,

$$\frac{d\sigma(pA \rightarrow h + X)}{dyd^2p_\perp} \Big|_{Ref.[1]} = \int q(x_p, \mu) \otimes D_q(z, \mu) \otimes \mathcal{F}_{Y_0}(r_\perp) \left[1 + \alpha_s \int_{Y_0}^Y dY' \text{BK} \otimes + \alpha_s (\text{other terms}) \right] , \quad (2)$$

for incoming quark channel contribution, where y and p_\perp are rapidity and transverse momentum of final state hadron, $q(x_p, \mu)$ and $D_q(z, \mu)$ are the integrated quark distribution from the proton and fragmentation function for the final state hadron, respectively. $\mathcal{F}_{Y_0}(r_\perp)$ is defined as the dipole amplitude from nucleus, and $\alpha_s(\text{other terms})$ represent those contributions which are not related to small- x evolution. Since we focus on the small- x factorization here, we omit those terms and simplify the formula for convenience. To obtain the differential cross section depending on transverse momentum, the Fourier transformation is performed. The above large logarithmic correction in terms

of $\alpha_s(Y - Y_0)$ ($Y - Y_0 = \ln(sx_g/m_p^2)$ with $Y_0 = \ln(1/x_g)$ and $x_g = p_\perp e^{-y}/z\sqrt{s}$) can be absorbed into the dipole amplitude $\mathcal{F}(r_\perp)$. Therefore, at the level of the leading logarithmic approximation, their result can be written as

$$\frac{d\sigma(pA \rightarrow h + X)}{dyd^2p_\perp} \Big|_{Ref.[1]} = \int q(x_p, \mu) \otimes D_q(z) \otimes \mathcal{F}_{Y=\ln(s/m_p^2)}(r_\perp) [1 + \alpha_s(\text{other terms})] , \quad (3)$$

where the α_s correction does not contain any large logarithms associated with small- x evolution. This clearly contradicts with the concept of factorization as well as Eq. (1), where leading order result depends on x_g , instead of $\ln(s/m_p^2)$ ¹. The above result also means that the dipole amplitude is independent of the rapidity of produced hadrons y . It would be a universal function only depending on total energy s , meaning that one only needs one single dipole amplitude for RHIC and LHC separately. The direct consequence is that the gluon saturation scale does not depend on the rapidity of the produced hadron either. On the other hand, the dipole amplitude $\mathcal{F}(r_\perp)$, which is derived from the scattering amplitude between the quark and the target nucleus, should not depend on the total centre of mass energy s .

In our opinion, the gluon rapidity $y_g \equiv \ln \frac{1}{1-\xi}$ ² in Ref. [1] by definition should be the rapidity separation between the radiated gluon and the parent *quark*, instead of the projectile *proton*. Only in very forward production, are these two quantity the same. But conceptually they differ by a factor of $\ln \frac{1}{x_p}$. Thus, if one defines $y_A = Y - y_g$ as the rapidity of the radiated gluon w.r.t. the target nucleus, Y should be the rapidity interval between the quark and target nucleus, which is, in principle, the same as $Y_0 \equiv \ln \frac{1}{x_g}$. Therefore, the new finite correction introduced in Ref. [1] should be identically zero.

As we have shown in Ref. [3], the choice of rapidity interval should reflect the correct leading logarithmic contribution from gluon radiation at small- x . Here, we elaborate in more details, and concentrate on large logarithms from small- x evolution from one gluon radiation. The kinematics is specified as follows: incoming quark with momentum $p = (p^+, 0^-, 0_\perp)$ scattering on the nucleus with momentum $P_A = (0^+, P_A^-, 0_\perp)$; final state quark $k = (\xi p^+, k^-, k_\perp)$ and gluon $k_1 = ((1-\xi)p^+, k_1^-, k_{1\perp})$. The rapidity divergence associated with small- x physics comes from the kinematic region of the radiated gluon parallel to the nucleus, leading to the following integral,

$$\frac{\alpha_s N_c}{2\pi} \int_0^1 \frac{d\xi}{(1-\xi)} . \quad (4)$$

This rapidity divergence appears in both real and virtual graphs. To regulate this divergence, a cut-off scheme can be used, which has to apply to both real and virtual contributions consistently. In particular, the cut-off in the above integral reflects the small- x logarithms, which can be explicitly identified from the kinematics. According to the on-shell kinematical requirement for the radiated gluon: $k_1^2 = 0$, one gets $k_1^- = \frac{k_{1\perp}^2}{2(1-\xi)p^+}$. Due to energy momentum conservation, we have strong constraints on $k_1^- < P_A^-$. Therefore, ξ -integral in the real diagram is constrained as

$$(1-\xi) > \frac{k_{1\perp}^2}{k_\perp^2} x_g (1 + \mathcal{O}(x_g)) , \quad (5)$$

where $\frac{k_{1\perp}^2}{2p^+P_A^-} = \frac{k_{1\perp}^2}{x_p s} = \frac{k_{1\perp}^2}{k_\perp^2} x_g$ is used to arrive at the above expression. Therefore, the rapidity divergent integral leads to the following large logarithm,

$$\frac{\alpha_s N_c}{2\pi} \ln \left(\frac{1}{x_g} \right) , \quad (6)$$

plus terms which are subleading in the small- x resummation, such as $\ln(k_\perp^2/k_{1\perp}^2)$. This is how the large logarithms emerge in the gluon radiation. Physically, the small- x evolution resum large logarithms coming from collinear gluon radiation with large rapidity difference while the transverse momentum is the same order. Therefore, the leading logarithms in this process is coming from $\ln(1/x_g)$. It is not in terms of $\ln(s/m_p^2)$. This is essentially the reason that the target gluon distribution function is function of x_g instead of function of $\ln(s/m_p^2)$. Similar analysis has also been applied to obtain the Sudakov double logarithms in hard processes in pA collisions [5], where the exact kinematics of

¹ In any application of factorization in hadronic process, the leading order perturbative calculation represents the leading logarithmic resummation, under the assumption that the factorization is valid and the relevant evolution equation is known. This applies to the case discussed here.

² $(1-\xi)$ is the longitudinal momentum fraction that the radiated gluon carries with respect to the parent quark.

the radiated gluon is key to derive the consistent resummation results. Of course, a complete factorization should allow us to have freedoms to choose the rapidity for the dipole amplitude, supplemented with the cancellation of this rapidity dependence in the final result. Technically, this can be done by choosing another cut-off (such as $(1 - \xi) > \delta = e^{-Y_\mu}$) for both the dipole amplitude and the cross section calculation. After performing the subtraction, the remaining hard part will depend on the difference between Y_μ and $Y_{phys} = \ln(1/x_g)$, whereas the dipole amplitude depending on Y_μ . In the final factorization formula, the Y_μ dependence cancels out. Therefore, Y_μ can be served as factorization scale for the rapidity factorization. We can do the same computation for any other scheme, such as tilting the Wilson line, and we should be able to obtain the same results. Therefore, the small- x factorization leads to the following expression for this part [3],

$$\frac{d\sigma(pA \rightarrow h + X)}{dyd^2p_\perp} \Big|_{Ref.[3,4]} = \int q(x_p, \mu) \otimes D_q(z, \mu) \otimes \mathcal{F}_{Y_\mu}(r_\perp) \left[1 + \alpha_s \int_{Y_\mu}^{Y_{phys}} dY' \text{BK} \otimes + \alpha_s (\text{other terms}) \right], \quad (7)$$

where Y_μ represents the scale separation for the small- x factorization. The above equation is physically different from Eq. (2) which was implied in Ref. [1].

In conclusion, we have pointed out that the leading logarithms identified in Ref. [1] is not correct and the claimed rapidity factorization is not consistent with the small- x factorization proposed in Ref. [3].

As a final note, we would like to emphasize that the factorization formula we proved in Ref. [3] is only valid in the small- x domain, where the saturation scale Q_s set the hard momentum scale. This factorization formula will break down at large transverse momentum $p_\perp > Q_s$. This is exactly what has been shown in the detailed numeric studies in Ref. [4], where it was found that the NLO corrections become negative in the transverse momentum region beyond the saturation scale Q_s for a given rapidity. This is because, in this region, the hard gluon radiation becomes dominant contribution, and thus it is appropriate to apply the collinear factorization to calculate the differential cross section, instead of using the small- x factorization formalism. Therefore, we shall expect a matching between low and high transverse momentum region for inclusive hadron production in pA collisions: in large transverse momentum ($> Q_s$) region, we apply collinear factorization; while in low transverse momentum ($< Q_s$) region, we apply small- x factorization, and in between, we should match between these two calculations. Following this idea, it is found that the transverse momentum spectrum data from low to high transverse momentum region in dAu collisions at RHIC can be described [6].

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